→ Product Rule

$$P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

→ Bayes Rule

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A) \cdot P(A)}{P(B)}$$

→ Chain Rule

$$P(A,B,C) = P(C|A,B) \cdot P(A,B) = P(C|A,B) \cdot P(B|A) \cdot P(A)$$

// different way to write the same

$$P(C|A \cap B)$$

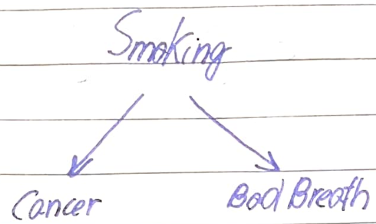
→ $$P(\text{Cause}/\text{Effect}) = \frac{P(\text{Effect}/\text{Cause})\, P(\text{Cause})}{P(\text{Effect})}$$

$$\text{cause} \rightarrow \text{Effect}$$

→ Naive Bayes (Conditional Independance)

$$P(\text{Cause}, \text{Effect}_1 \ldots \text{Effect}_n) = P(\text{cause}) \prod_i P(\text{Effect}_i/\text{cause})$$

1.



P(Smoking) = 0.2
P(cancer / smoking) = 0.6
P(Bad Breath / smoking) = 0.95

P(Smoking, Cancer, Bad Breath) = P(Smoking) · ....

... P(Cancer / Smoking) · P(Bad Breath / Smoking) =

0.2 × 0.6 . 0.95

2. Noisy - Or

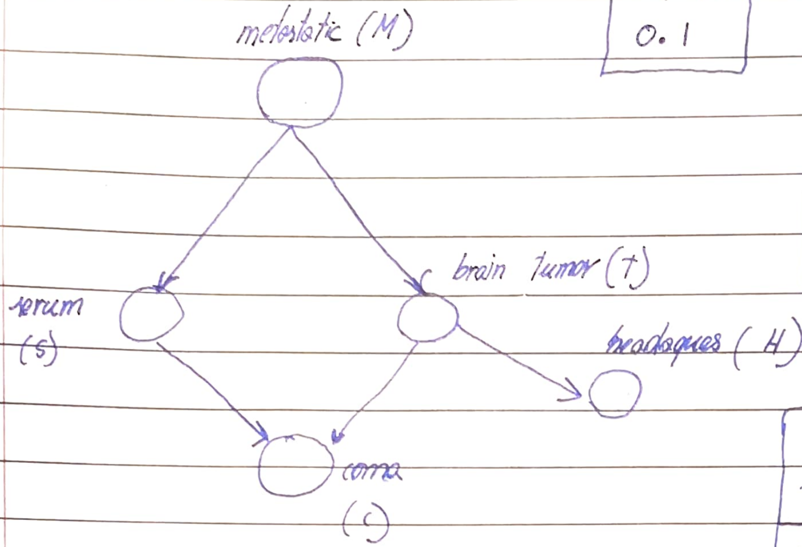
→ Independent $\boxed{\text{FAILURE}}$ probability

P(Late Start	P(Ignore	P(Fail	P(¬ Fail
F	F	1	0
F	T	0.7	0.3
T	F	0.8	0.2
		1-0.06 = 0.94	0.3×0.2 = 0.06

3.

No Bayesian Network → $2^5 - 1 = 31$
Bayesian Network → 1 + 2 + 2 + 4 + 2 = 11 (Check Network

q0



P(M)
0.1

M	P(S|M)
T	0.8
F	0.2

M	P(T|M)
T	0.7
F	0.1

S	T	P(C|S,T)
T	T	0.95
T	F	0.85
F	T	0.85
F	F	0.01

T	P(H|T)
T	0.9
F	0.7

$P(m, \neg t, h, s, \neg c) \rightarrow ?? \Rightarrow$ calculate it using Bayesian Nets

$\parallel$

$P(h/\neg t) \times P(\neg c/s, \neg t) \times P(\neg t/m) \times P(s/m) \times P(m)$

$\quad 0.7 \qquad\qquad 0.15 \qquad\qquad 0.3 \qquad\qquad 0.8 \qquad\qquad 0.1$

$= 0.00252$

5° Calculation of probabilities using enumeration.

$$P(M/h,s) = \frac{P(M,h,s)}{P(h,s)} = \alpha\, P(M,h,s)$$

Fixed Variables → h, s (These ones are always true)
Variables with impact on Fixed variables → t, c (hidden)
Variable we want to Know → m

$$\sum_T \sum_C P(M,h,s,t,c) =$$

$$= \sum_T \sum_C \underbrace{P(M) \times P(s/h)}_{} \times P(h/t) \times P(t/m) \times P(c/s,t)$$

These two are out
because the hidden variable do not have impact on them

$$= P(M) \times P(s/m) \times \sum_T \sum_C P(h/T) \times P(t/m) \times P(c/s,t)$$

$$\alpha \begin{pmatrix} P(m) \times P(s/m) \times \sum_T \sum_C P(h/t) \times P(T/m) \times P(c/s,t) \\ P(\neg m) \times P(s/\neg m) \times \sum_T \sum_C P(h/t) \times P(T/m) \times P(c/s,t) \end{pmatrix}$$

$\underset{0.1}{P(m)} \times \underset{0.8}{P(s/m)} \times$

- t and c True: $\underset{0.9}{P(h/t)} \times \underset{0.7}{P(t/m)} \times \underset{0.95}{P(c/s,t)}$
- t and c False: $\underset{0.7}{P(h/\neg t)} \times \underset{0.3}{P(\neg t/m)} \times \underset{0.15}{P(\neg c/s,\neg t)}$
- t is true and c False: $\underset{0.9}{P(h/t)} \times \underset{0.7}{P(t/m)} \times \underset{0.05}{P(\neg c/s,t)}$
- t is False and c true: $\underset{0.7}{P(h/\neg t)} \times \underset{0.3}{P(\neg t/m)} \times \underset{0.85}{P(c/s,\neg t)}$

→

$$P(m) \times P(s/m) \times \sum_{t}\sum_{c} P(h/t) \times P(c/s,t) \times P(t/m) =$$

$$0.1 \times 0.8(0.9 \times 0.7 \times 0.95 + 0.9 \times 0.7 \times 0.05 + 0.7 \times 0.3 \times 0.85 + 0.7 \times 0.3 \times 0.15) = 0.0672$$

$$P(\neg m) \times P(s/\neg m) \times \sum_{t}\sum_{c} P(h/t) \times P(c/s,t) \times (t/\neg m)$$

$$0.9 \times 0.2 \times (\underbrace{0.9 \times 0.1 \times 0.95}_{T \text{ and } C} + \underbrace{0.9 \times 0.1 \times 0.05}_{T \text{ and } \neg C} + \underbrace{0.7 \times 0.9 \times 0.85}_{\neg t \text{ and } c} + \underbrace{0.7 \times 0.9 \times 0.15}_{\neg t \text{ and } \neg c}) = 0.1296$$

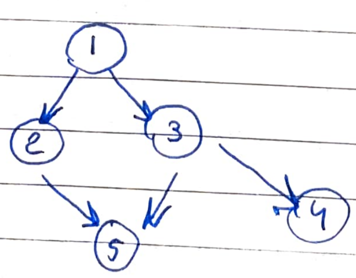
$$P(M/h,s) = \alpha \begin{pmatrix} 0.0672 \\ 0.1296 \end{pmatrix} = \begin{pmatrix} 0.34 \\ 0.66 \end{pmatrix}$$

So calculate probability with prior Sampling

Joint probabilities of $P(m, s, t, h, c)$

Random numbers = [0.14, 0.57, 0.01, 0.43, 0.59, 0.50, 0.12, 0.54, 0.97, 0.51, 0.49, 0.67, 0.96, 0.55, 0.89, 0.21, 0.34]

Order :



→

6°

Round 1

→ $0.14 \Rightarrow P(m) = 0.10 \Rightarrow \neg m$

→ $0.57 \Rightarrow P(s|\neg m) = 0.2) \Rightarrow \neg s$

→ $0.01 \Rightarrow P(t|\neg m) = 0.1) \Rightarrow t$

→ $0.43 \Rightarrow P(h|t) = 0.9) \Rightarrow h$

→ $0.59 \Rightarrow P(c|\neg s, t) = 0.85 \Rightarrow c$

1^{st} Round = $(\neg m, \neg s, t, h, c)$
2^{nd} Round = $(\neg m, s, \neg t, \neg h, c)$
3^{rd} Round = $(\neg m, \neg s, \neg t, h, \neg c$
4^{th} Round = $(\neg m, \neg s, \neg t, h, \neg c)$
5^{t} Round = $(\neg m, \neg s, \neg t, h, c)$

5 Rounds

$$P(\neg m, \neg s, t, h, c) = \frac{1}{5}$$

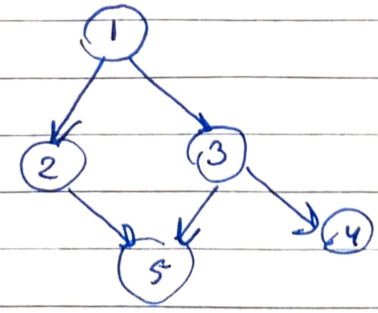
$$P(\neg m, s, \neg t, \neg h, c) = \frac{1}{5}$$

$$P(\neg m, \neg s, \neg t, h, \neg c) = 0.6 = \frac{3}{5}$$

7° Calculating P with Rejection sampling

$P(m|h,s) \Rightarrow$ Only choose samples where h and s are True

→ Choose order



7^{th}

Choose order ..... h and s → True

Round 1

→ 0.14 → $P(m) = 0.10 \Rightarrow \neg m$

→ 0.57 → $P(s|\neg m) = 0.2 \rightarrow \neg s$

¡¡ START AGAIN !! → s is False

Round 2

→ $0.01 = P(m) = 0.10 \Rightarrow m$

→ $0.43 = P(s|m) = 0.8 \Rightarrow s$ → Pass

→ $0.59 = P(t|m) = 0.7 \Rightarrow t$

→ $0.50 = P(h|t) = 0.9 \Rightarrow h$ → Pass

→ $0.12 = P(c|t,s) = 0.9 \Rightarrow c$

Round 3 → $(\neg m, s, \neg t, h, c)$

Round 4 → $(\neg m, s, \neg t, h, c)$

$P(m, s, t, h, c) = 1/3$

$P(\neg m, s, \neg t, h, c) = 2/3$

→

9th Exercise

Importance sampling →

$P(m / h, s) \Rightarrow$

s and h are True by definition ⇒
we make sure this happens

→ 0.14 → $P(m) = 0.10$ → $\neg m$
→ $P(s \mid \neg m) = 0.2$ → $w = 0.2$
→ 0.57 ⇒ $P(t \mid \neg m) = 0.1$ → $\neg t$
→ $P(h \mid \neg t) = 0.7$ → $w = 0.2 \times 0.7 = 0.14$
→ 0.01 ⇒ $P(c \mid \neg t, s) = 0.85$ ⇒ c

$$\underbrace{P(\neg m, h, \neg t, h, c)}_{} = 0.14$$

However, this cannot be used to calculate $P(m / h, s)$ because m is False in this sample.
We have to sample a case where m is True.

↓

Let imagine that we reach the sample that starts with a 0.05 as a random number.

→ Sample 0.05 → $P(m) = 0.10$ → m
→ $P(s \mid m) \Rightarrow 0.8$ ⇒ $w = 0.8$
→ Sample 0.34 ⇒ $P(t / m) = 0.7$ → t
→ $P(h / t) = 0.9$ ⇒ $w = 0.8 \times 0.9 = 0.72$
→ c is true

$$P(m, s, t, h, c) = 0.72$$

9Th Exercise

$P(m/h,s)$ with Gibbs sampling

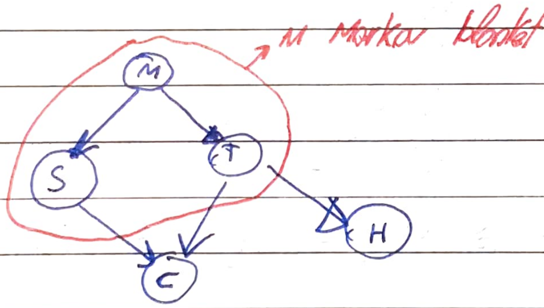
Fixed variables h, s

→ Assumption →

The values of M, T, C are equaly likely ≈50%

→ Sample 1 0.14 → $P(M) = 0.50$ = m
→ Sample 2 0.57 → $P(t) = 0.50$ = $\neg t$
→ Sample 3 0.01 → $P(C) = 0.50$ = c

→ Firs ve use M and its Markov Blanket.



$$P(M/s,\neg t) = \alpha P(M) \odot P(s/M) \odot P(\neg t/M)$$

$$\alpha \begin{pmatrix} 0.1 \times 0.8 \times 0.3 \\ 0.9 \times 0.2 \times 0.9 \end{pmatrix} \begin{matrix} \to m \\ \to \neg m \end{matrix} \Rightarrow$$

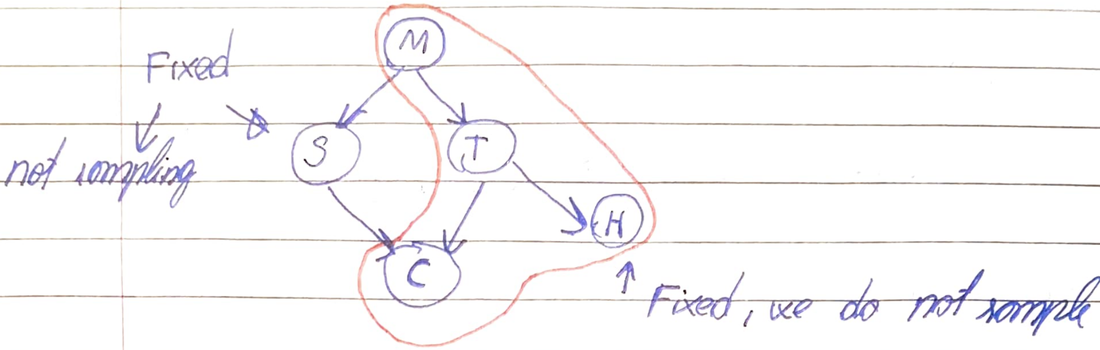
$$\alpha \begin{pmatrix} 0.024 \\ 0.162 \end{pmatrix} = \begin{pmatrix} 0.13 \\ 0.87 \end{pmatrix}$$

→ Sample 0.43 → $P(m) = 0.13$ → $\neg m$ ⇒

$(\neg m, s, \neg t, c, h)$

Next variable is T

Markov Blanket of T



$P(T \mid \text{markov blanket}(T)) = \alpha\, P(T \mid \neg m) \times P(c \mid s, T) \times P(h \mid T)$

$$\begin{pmatrix} 0.1 \times 0.9 \times 0.95 \\ 0.9 \times 0.7 \times 0.85 \end{pmatrix} = \begin{pmatrix} 0.0855 \\ 0.5355 \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} 0.14 \\ 0.86 \end{pmatrix}$$

$\rightarrow$ Sample $0.59 \rightarrow P(t) = 0.14) \Rightarrow \neg t \Rightarrow$

$(\neg m, s, \neg t, c, h)$

So we repeat this process, and we count how many times a state with m appears.